

September 23, 2016

Soft See-Saw: Radiative Origin of Neutrino Masses in SUSY Theories

Luka Megrelidze¹ and Zurab Tavartkiladze²

Center for Elementary Particle Physics, ITP, Ilia State University, 0162 Tbilisi, Georgia

Abstract

Radiative neutrino mass generation within supersymmetric (SUSY) construction is studied. The mechanism is considered where the lepton number violation is originating from the soft SUSY breaking terms. This requires MSSM extensions with states around the TeV scale. We present several explicit realizations based on extensions either by MSSM singlet or $SU(2)_w$ triplet states. Besides some novelties of the proposed scenarios, various phenomenological implications are also discussed.

Keywords: Lepton number violation; Neutrino masses; supersymmetry; radiative corrections.

1 Introduction

One of the missing piece of the Standard Model (SM) is the consistent neutrino sector required for accommodation of the neutrino data [1]. Extensions based on type I [2], type II [3] and type III [4] see-saw mechanisms have been suggested, which at tree level induce effective dimension five ($d = 5$) $\Delta L = 2$ lepton number violating operator [5]

$$\frac{\lambda_{ij}}{M_*} l_i l_j H H , \quad (1)$$

where l_i ($i, j = 1, 2, 3$ are family indices) and H are SM lepton and Higgs doublets respectively. The (1) type couplings, in turn, generate neutrino masses and mixings after EW symmetry breaking.

It was shown [6–9] that, augmenting the SM by specific states and couplings, the operators of Eq. (1) can be generated radiatively at one (or higher) loop level. This possibility, referred as radiative neutrino mass generation mechanism, offers many interesting scenarios with rich phenomenological implications [6–9]. It is a curious fact that, besides some studies [10], within SUSY constructions

¹E-mail: luka.megrelidze.1@iliauni.edu.ge

²E-mail: zurab.tavartkiladze@gmail.com

such possibilities have not been pursued much.³ In this work, aiming to fill this gap and address this issue within SUSY scenarios, we offer possibilities of loop induced neutrino masses, where lepton number violation occurs in soft SUSY breaking couplings and is transferred in the SM neutrino sector at the loop level. Referring to this possibility as ‘soft see-saw’ mechanism, we present several extensions which naturally realize this programme. The following three types of extensions are considered: *i*) extension with MSSM singlet (matter) superfields, *ii*) extension with a pair of $SU(2)_w$ triplet-antitriplet scalar superfields carrying $U(1)_Y$ hypercharges ± 2 , and *iii*) extension with matter $SU(2)_w$ triplet and $U(1)_Y$ neutral superfields. We call these three scenarios soft type I, soft type II and soft type III see-saw scenarios respectively. We work out details of each construction and discuss phenomenological implications.

The paper is organized as follows. In the next section we discuss lepton number violating operators of different dimensions, involving MSSM states and list some of them. Then, as a demonstration, we compute the $d = 5$, $\Delta L = 2$ operator of type (1), induced via 1-loop due to presence of a quartic ($d = 4$) $\Delta L = 2$ operator. In Sect. 3 we present models - extensions of the MSSM - where the lepton number violation takes place in soft SUSY breaking terms and show how integrating out the extra states give effective $\Delta L = 2$ operators. We consider extensions with MSSM singlet matter (right handed neutrino) superfields and with $SU(2)_w$ triplet states. Based on these extensions different scenarios (e.g. type I, type II-A, type II-B and type III soft see-saw) emerge. In each case neutrino masses are induced via loops. Corresponding 1-loop and 2-loop results are presented. Sect. 4 contains discussion and outlook with some prospects for future studies. In appendix A we present supergravity formalism for calculation of soft SUSY breaking terms emerged from hidden sector superpotential and non-minimal Kähler potential couplings. We give the details of the SUSY breaking via Polonyi superpotential and specific non-minimal Kähler potential - the system insuring for SUSY breaking X superfield $F_X \sim m_{3/2} M_{Pl}$ (where $m_{3/2}$ is a gravitino mass) and adequately suppressed value for $\langle X \rangle$. Both these are needed for our model building. In appendix B detailed derivations of effective $\Delta L = 2$ quartic couplings, emerging by integration of MSSM singlet and $SU(2)_w$ triplet states, are given. In appendix C we present the calculation of the 2-loop contribution to the neutrino masses.

2 Some $\Delta L = 2$ Lepton Number Violating Operators with MSSM States

The lepton sector of the MSSM involves the following D and F -term couplings:

$$(l^\dagger e^V l + e^{c\dagger} e^V e^c)_D \rightarrow \tilde{l}^* \tilde{V} l + \tilde{e}^{c*} \tilde{V} e^c + \text{h.c.} \quad (2)$$

$$(Y_E l e^c h_d)_F + \text{h.c.} \rightarrow Y_E \left(l e^c h_d + \tilde{l} e^c \tilde{h}_d + l \tilde{e}^c \tilde{h}_d \right) + \text{h.c.} \quad (3)$$

where schematically V indicates all appropriate gauge superfields multiplied by proper generator and gauge coupling. Without soft SUSY breaking terms, it is possible to define the lepton number

³Note that, like within SM, also in minimal supersymmetric extension of the SM (MSSM), it is hard to see how neutrino masses $\gtrsim 0.05$ eV can be generated even with non-renormalizable [i.e. the cut off scale $M_* = M_{Pl}$ in Eq. (1)] interactions are taken into account.

(L -numbers for l, e^c) and also slepton number (\tilde{L} -numbers for \tilde{l}, \tilde{e}^c) separately.⁴ Thus, the couplings in (2) and (3) possess two independent symmetries. However, including soft SUSY breaking terms - the gaugino masses $\frac{1}{2}M_{\tilde{V}_a}\tilde{V}_a\tilde{V}_a$ - give the slepton numbers fixed equal to the corresponding lepton numbers: $\tilde{L} = L$.⁵ Thus, all components from a given superfield have same lepton number. Therefore, if for instance, in the soft SUSY breaking sector the lepton number will be broken, it will be transferred in the fermion sector via radiative corrections.

In our consideration we assume that the whole Lagrangian respects the matter parity, under which states transform as $\Phi \rightarrow (-1)^{3(B-L)+2S}\Phi$, where B and L are baryon and lepton numbers respectively and S indicates the spin of the Φ state. This symmetry insures that no baryon and lepton number violating couplings present at renormalizable level and LSP is a stable.

Considering higher dimensional operators, in SUSY (with matter parity) the lepton number violation starts from the F -term $d = 5$ operator

$$\int d^2\theta (lh_u)^2 + \text{h.c.} \rightarrow \left((ll)h_uh_u + 2(\tilde{l}h_u)\tilde{l}h_u + (\tilde{h}_u\tilde{h}_u)\tilde{l}\tilde{l} \right) + \text{h.c.} \quad (4)$$

which is the SUSY analog of the (1) coupling. At r.h.s of Eq. (4) l denotes SM fermionic lepton and h_u is up type Higgs doublet. Symbols with tildes denote their superpartners respectively. The coefficients at r.h.s of Eq. (4) are related due to SUSY. However, in general they can be different especially if some of these couplings emerge via SUSY breaking terms. Therefore, we write each of them with independent coefficients:

$$d = 5, \quad \Delta L = 2 \quad \text{operators :} \quad \lambda_5(lh_u)^2 + \lambda'_5(\tilde{l}\tilde{h}_u)^2 + \lambda''_5\tilde{l}h_u l\tilde{h}_u + \text{h.c.} \quad (5)$$

λ_5 -type couplings are directly constrained from the neutrino masses. The λ'_5 and λ''_5 operators induce λ_5 -term via loops in which soft SUSY breaking couplings participate. Therefore, constraints on λ'_5 , λ''_5 will depend on SUSY spectroscopy. The λ'_5 operator emerges in one of the model (named as type II-B soft see-saw model) we present in Sect. 3.2.

Before going to the higher dimensional operators, note that with MSSM states one can write $\Delta L = 2$ quartic ($d = 4$) coupling

$$d = 4, \quad \Delta L = 2 \quad \text{operator :} \quad \lambda_4(\tilde{l}h_u)^2 + \text{h.c.} \quad (6)$$

This term is non-supersymmetric operator and to discuss its origin one needs to have some UV completion. In Sect. 3 we present models, where this operator emerges and compute λ_4 coupling in terms of model parameters.

Let us also give some $d = 6$, $\Delta L = 2$ couplings involving states from l and h_u superfields:

$$d = 6, \quad \Delta L = 2 \quad \text{operators :} \quad \lambda_6(l\tilde{h}_u)^2 + \lambda'_6(\tilde{l}\tilde{l})g\tilde{V}(\tilde{h}_uh_u) + \text{h.c.} \quad (7)$$

where in λ'_6 -term under $g\tilde{V}$ we assume $g_1\tilde{V}_1$ or $g_2\tilde{V}_2$ (with appropriate contraction of the $SU(2)_w$ gauge indices). This coupling emerges in type II-B soft see-saw model (discussed in Sect. 3.2).

Other $\Delta L = 2$ couplings can be obtained from the operators in Eqs. (5)-(7) by the substitution $h_u \rightarrow h_d^\dagger$. Also, the variety of $\Delta L = 2$ operators involving e^c and \tilde{e}^c states can be constructed.

⁴Provided that the gauginos \tilde{V}_a and the higgsinos \tilde{h}_d, \tilde{h}_u carry appropriate lepton and slepton charges.

⁵The relation $\tilde{L} = L$ also emerges if, instead of gaugino masses, the trilinear A -terms $A_E\tilde{l}\tilde{e}^ch_d$ are included.

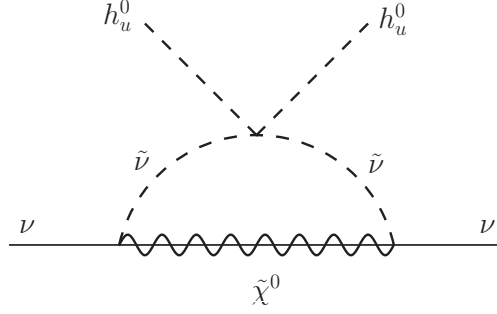


Figure 1: Diagram(s) responsible for neutrino masses via λ_4 -coupling.

The models with their generation would be interesting to consider, but in this work we pursue only specific constructions.

$\Delta L = 2$ operators, including those we gave above, by radiative corrections will be converted to the operator $\lambda_5(lh_u)^2$ - responsible for the neutrino mass generation. In the next subsection we give details of the calculation of this λ_5 -term emerged from the λ_4 coupling of Eq. (6) via gaugino/higgsino loop dressings.

2.1 $\Delta L = 2$ Quartic Coupling and Neutrino Mass

The coupling (6) is invariant under SM gauge symmetry. Since we are interested in neutrino mass generation, we extract from it the neutral components:

$$\lambda_4(\tilde{\nu}h_u^0)^2 + \text{h.c.} \quad (8)$$

The combination $\tilde{\nu}\tilde{\nu}$ will be converted to $\nu\nu$ by the neutralino dressing diagram(s) shown in Fig. 1. The relevant terms are also

$$i\frac{g_1}{2}\tilde{B}\tilde{l}^\dagger l - i\frac{g_2}{\sqrt{2}}\tilde{W}_b^a\tilde{l}_a^* l^b + \text{h.c.} \rightarrow \frac{i}{2}\nu\left(g_1\tilde{B} - g_2\tilde{W}^0\right)\tilde{\nu}^* + \text{h.c.} \quad (9)$$

(given in 2-component notations of Ref. [11]) where \tilde{B} and \tilde{W}_b^a are $U(1)_Y$ and $SU(2)_w$ gauginos respectively. The indices $a, b = 1, 2$ belong to the $SU(2)_w$ gauge group. \tilde{W} 's matrix is defined as

$$\tilde{W}_b^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \tilde{W}^0 & \sqrt{2}\tilde{W}^+ \\ \sqrt{2}\tilde{W}^- & -\tilde{W}^0 \end{pmatrix}_b^a. \quad (10)$$

We are looking for the operator:

$$\frac{1}{2}\kappa_{ij}^\nu\nu_i\nu_j(h_u^0)^2 + \text{h.c.} \quad (11)$$

where κ_{ij}^ν will be computed in terms of couplings appearing in (8), (9), together with other model parameters given below.

By diagonalization of 4×4 neutralino mass matrix (of the states $\{-i\tilde{B}, -i\tilde{W}^0, \tilde{h}_d^0, \tilde{h}_u^0\}$), the \tilde{B} and \tilde{W}^0 get transformed as

$$\tilde{B} = U_{\chi^0}^{1\bar{a}} \tilde{\chi}_{\bar{a}}^0, \quad \tilde{W}^0 = U_{\chi^0}^{2\bar{a}} \tilde{\chi}_{\bar{a}}^0, \quad (\bar{a} = 1, 2, 3, 4), \quad (12)$$

where $\chi_{\bar{a}}^0$ is a physical mass eigenstate neutralino with mass $M_{\chi^0}^{\bar{a}}$. Using these in Eq. (9), the relevant couplings will be

$$\frac{i}{2}\nu (g_1 U_{\chi^0}^{1\bar{a}} - g_2 U_{\chi^0}^{2\bar{a}}) \chi_{\bar{a}}^0 \tilde{\nu}^* + \text{h.c.} \quad (13)$$

Interactions in Eqs. (8), (13) together with neutralino Majorana type mass terms, generate the operator of Eq. (11) via 1-loop diagram(s) shown in Fig. 1. Upon evaluation of the loop integral for the expression of κ^ν we obtain:

$$\kappa_{ij}^\nu = -\frac{\alpha_2}{8\pi} (\lambda_4)_{ij} \sum_{\bar{a}=1}^4 \frac{(\tan \theta_W U_{\chi^0}^{1\bar{a}} - U_{\chi^0}^{2\bar{a}})^2}{M_{\chi^0}^{\bar{a}}} f(x_{\bar{a}}^i, x_{\bar{a}}^j), \quad (14)$$

where $\tan \theta_W = \frac{g_1}{g_2}$ and

$$x_{\bar{a}}^i = \left(\frac{m_{\tilde{\nu}_i}}{M_{\chi^0}^{\bar{a}}} \right)^2, \quad f(x_1, x_2) = \frac{x_1 \ln x_1 - x_2 \ln x_2 - x_1 x_2 \ln \frac{x_1}{x_2}}{(x_1 - x_2)(x_1 - 1)(x_2 - 1)}. \quad (15)$$

From these expressions we can see that in order to have adequately suppressed neutrino masses ($\lesssim 0.1$ eV), for the SUSY particle masses \sim TeV, we need to have $\lambda_4 \lesssim 10^{-8}$. In the next section we will see that this suppression can be naturally realized within presented models.

3 Soft See-Saw

In this section we present models which generate some of the $\Delta L = 2$ operators [in particular: λ_4, λ'_5 and λ'_6 -type couplings of Eqs. (6), (5) and (7) resp.]. This happens through the soft SUSY breaking sector. Therefore, we refer to it as a soft see-saw mechanism. Scenarios we consider are based on extensions either with MSSM singlet matter superfields⁶, or with a pair of $SU(2)_w$ triplet-antitriplet scalar superfields, or on a model with matter $SU(2)_w$ triplet superfields. Each case is investigated separately.

3.1 Type I Soft See-Saw

In this case, the MSSM is extended with right handed neutrino (RHN) superfields N . Since within our scenario, the masses of these states are near TeV scale, in order to avoid unacceptably large neutrino masses, we will need to suppress the $Y_\nu l N h_u$ type Yukawa superpotential couplings. Instead demanding ad hoc condition $|Y_\nu| \lesssim 10^{-7}$, we will forbid such superpotential coupling by the R -symmetry, under which the superfield ϕ_i and the superpotential transform as:

$$\phi_i \rightarrow e^{iR_i} \phi_i, \quad W \rightarrow e^{i\omega} W. \quad (16)$$

⁶This kind of extension, with lepton number violating soft SUSY breaking terms, has been considered in Refs. [10]. Here we give detailed discussion of the setup, necessary ingredients, generation of effective λ_4 coupling and some constraints.

	W	X	h_u	N	l
R	ω	1	α	$\frac{1}{2}$	$-\frac{1}{2} - \alpha$
	h_d	e^c	q	u^c	d^c
R	$1 - \alpha$	$\omega + 2\alpha - \frac{1}{2}$	α_q	$\omega - \alpha_q - \alpha$	$\omega - \alpha_q + \alpha - 1$

Table 1: R charges in the Type I soft see-saw model. $\omega \neq m + 1/2$ ($m \in \mathbb{Z}$).

With the R charge assignment for the superpotential W and superfields l, h_u, N given in Table 1, together with the couplings $lN h_u$, also the NN -type superpotential terms are forbidden. However, as we will see shortly, the lepton number violating soft SUSY breaking terms will be induced. These type of terms will come from the Kähler potential, via the SUSY breaking. The latter will occur in a hidden sector with MSSM singlet superfield X having non-zero F -term $F_X \sim m_{3/2} M_{Pl}$. The R charge of the X is selected to be $R_X = 1$. In Table 1 we summarize R charges of all superfields to be considered. With this assignment, the following minimal and non-minimal Kähler (denoted below as \mathcal{K}_m and \mathcal{K}_{nm} respectively) potential couplings are allowed:

$$\mathcal{K}_m = \sum_f f^\dagger e^{g_a V_a} f, \quad \mathcal{K}_{nm} = \frac{X^\dagger}{2M_{Pl}} \kappa N N + \frac{X^\dagger X}{\bar{M}^3} \kappa_A l N h_u + \frac{X^\dagger}{M_{Pl}} \kappa_h h_u h_d + \text{h.c.} \quad (17)$$

(On the second term in \mathcal{K}_{nm} , with cut off scale $\bar{M} < M_{Pl}$, we comment below.) With these, the generated soft SUSY breaking terms involving \tilde{N} will be:

$$V(\tilde{N}) = \tilde{l}^T A_N \tilde{N} h_u + \frac{1}{2} \tilde{N}^T B_N \tilde{N} + \text{h.c.} + \tilde{N}^\dagger M_N^2 \tilde{N}, \quad (18)$$

where $\tilde{l}^T = (\tilde{l}_1, \tilde{l}_2, \tilde{l}_3)$ and $\tilde{N}^T = (\tilde{N}_1, \tilde{N}_2, \dots)$ (the number of \tilde{N} states should be ≥ 2). The matrix M_N^2 is hermitian while B_N is a symmetric: $M_N^2 = (M_N^2)^\dagger$ and $B_N^T = B_N$. The mass² terms in (18) with $F_X \sim m_{3/2} M_{Pl}$ are $M_N^2 \sim |F_X|^2 / M_{Pl}^2 \sim m_{3/2}^2$. The soft B -terms are $B_N \sim \kappa |F_X|^2 / M_{Pl}^2 \sim \kappa m_{3/2}^2$, while with $\bar{M} \sim (m_{3/2} M_{Pl}^2)^{1/3}$ the trilinear A -term will be $A_N \sim \kappa_A |F_X|^2 / \bar{M}^3 \sim \kappa_A m_{3/2}$. For detailed discussion and derivations see Appendix A. In the same Appendix we discuss the generation of F_X via Polonyi superpotential and specific Kähler potential for X .

From the couplings in Eq. (18), by integrating out the \tilde{N} states we get the operator of Eq. (6). In particular, by assumption $\kappa \lesssim 1/3$, $\kappa_A v_u \lesssim 0.3 m_{3/2}$, the expression for λ_4 can be approximated as:

$$\lambda_4 \simeq \frac{1}{2} A_N \frac{1}{M_N^2} B_N^\dagger \frac{1}{(M_N^2)^T} A_N^T. \quad (19)$$

Corresponding diagram is given in Fig. 2. Derivation and more accurate expression for λ_4 is given in Appendix B by Eq. (B.7). From the structure of (19) and process given in Fig. 2 it is clear why it is fair to call this mechanism (for generating of λ_4 coupling) the type I soft see-saw. The quartic operator of (6) is forbidden in the SUSY limit, but is generated due to SUSY breaking $\Delta L = 2$ terms via integrating out the \tilde{N} states. With these, as was shown in section 2.1, the neutrino masses are generated at 1-loop level. The needed suppression for the λ_4 is easily obtained. For instance, with the SUSY particle masses $\sim \text{few} \cdot \text{TeV}$ and $\frac{A_N}{M_N} \sim \frac{\sqrt{B_N}}{M_N} \sim 10^{-2}$, from (19) we can get $\lambda_4 \sim 10^{-8}$ - guaranteeing suppressed values of the neutrino masses (~ 0.1 eV).

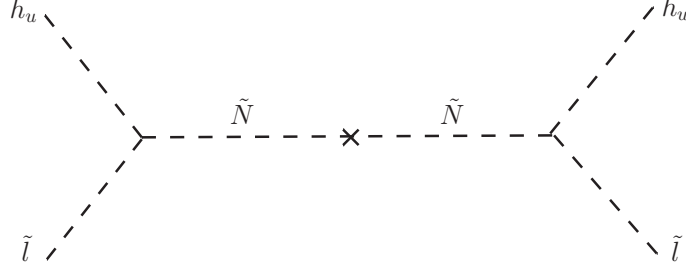


Figure 2: Diagram generating $(\tilde{l}h_u)^2$ term via \tilde{N} exchange.

The second operator in \mathcal{K}_{nm} of Eq. (17) can be obtained by integrating some heavy states. Here we give one example. By introducing additional MSSM singlet states $\mathcal{N}, \overline{\mathcal{N}}, \mathcal{N}'$ and $\overline{\mathcal{N}'}$ we can have the Kähler coupling $\mathcal{K}_{\mathcal{N}} = \frac{\kappa_{\mathcal{N}} X^\dagger}{M_{Pl}} \mathcal{N} \overline{\mathcal{N}}$ and the superpotential terms: $W_{\mathcal{N}} = M_{\mathcal{N}} \mathcal{N} \overline{\mathcal{N}} + \lambda_X X \mathcal{N} \overline{\mathcal{N}'} + M'_{\mathcal{N}} \mathcal{N}' \overline{\mathcal{N}'} + \lambda_{\mathcal{N}} \overline{\mathcal{N}'} l h_u$. One can easily verify that integration of the states $\mathcal{N}, \overline{\mathcal{N}}, \mathcal{N}', \overline{\mathcal{N}'}$ induces the Kähler operator $\frac{\kappa_{\mathcal{N}} \lambda_X \lambda_{\mathcal{N}}}{M_{Pl} M_{\mathcal{N}} M'_{\mathcal{N}}} X^\dagger X l N h_u$. Comparing this with Eq. (17), one can identify $\frac{k_A}{M^3} = \frac{\kappa_{\mathcal{N}} \lambda_X \lambda_{\mathcal{N}}}{M_{Pl} M_{\mathcal{N}} M'_{\mathcal{N}}}$.

As shown in Appendix A, by the construction one can insure that the VEV of the X field can be adequately suppressed. With $\langle X \rangle \lesssim 10^{-7} M_{Pl}$ the neutrino Dirac Yukawa couplings, generated via the Kähler potential will be $Y_\nu \lesssim 10^{-7}$ (see expression in (A.23) and related discussion before and after of this equation), i.e. not relevant for the neutrino masses. On the other hand, there is a low bound on the values of Y_ν couplings. They should be sizable enough to insure decays of the fermionic RHN states N within $\lesssim 0.3$ sec. in order to not affect the standard Big Bang Nucleosynthesis. With $M_N > M_l + M_{h_u}(M_{\tilde{l}} + M_{\tilde{h}})$ we will have decays $N \rightarrow l h_u(\tilde{l} \tilde{h})$, with the lifetime given by

$$\Gamma_{N \rightarrow l h_u(\tilde{l} \tilde{h})}^{-1} \approx \frac{8\pi}{|Y_\nu|^2 M_N} = 0.3 \text{ sec.} \times \left(\frac{2.34 \cdot 10^{-13}}{|Y_\nu|} \right)^2 \left(\frac{1 \text{ TeV}}{M_N} \right). \quad (20)$$

Since $Y_\nu \sim \langle X \rangle / M_{Pl}$, we will have the low bound on the VEV $\langle X \rangle \gtrsim 5.6 \cdot 10^5 \text{ GeV}$. Therefore, summarizing all above, we will have the following range:

$$2.34 \cdot 10^{-13} \lesssim Y_\nu \lesssim 10^{-7} \quad \longrightarrow \quad 5.6 \cdot 10^5 \text{ GeV} \lesssim \langle X \rangle \lesssim 2 \cdot 10^{11} \text{ GeV}. \quad (21)$$

The mass M_N is generated from the first coupling of \mathcal{K}_{nm} of (17), with the value $M_N \sim \kappa F_X / M_{Pl} \sim \kappa m_{3/2}$. Note that, it is easy to satisfy $M_N > M_l + M_{h_u}(M_{\tilde{l}} + M_{\tilde{h}})$ insuring the decays described in (20).

Closing this subsection, with the R -charge assignment given in Table 1, the MSSM Yukawa superpotential couplings are:

$$W_Y^{MSSM} = Y_U q u^c h_u + Y_D q d^c h_d + Y_E l e^c h_d. \quad (22)$$

Note that the direct $h_u h_d$ superpotential term is forbidden. However, as shown first in Ref. [12], the last coupling in (17) generates $\mu \sim \kappa_h m_{3/2}$ and $B\mu \sim \kappa_h m_{3/2}^2$ terms.

	X	h_u	Σ	$\bar{\Sigma}$	l
R	1	α	$1 + 2\alpha$	-2α	$-\frac{1}{2} - \alpha$
	h_d	e^c	q	u^c	d^c
R	$1 - \alpha$	$\omega + 2\alpha - \frac{1}{2}$	α_q	$\omega - \alpha_q - \alpha$	$\omega - \alpha_q + \alpha - 1$

Table 2: R charges in the Type II-A soft see-saw model, with $R_W = \omega$. By $\omega \neq m + 1/2$ ($m \in \mathbb{Z}$) the matter parity is automatic.

With the R -charge assignments given in Table 1 the lepton number violating superpotential couplings $X^n h_u l$ (which break also matter parity) are all forbidden for $\omega \neq m + 1/2$ (where m is an integer). Also the trilinear lepton number violating interactions $q d^c l$ and $e^c l l$ will be forbidden in the superpotential. The baryon number violating term $u^c d^c d^c$ can be easily avoided by specific selection of the phases α_q and α . By similar selection, one can make sure that matter parity violating terms do not emerge also from the Kähler potential.

3.2 Type II Soft See-Saw

In this case we extend the MSSM by introducing the pair of $SU(2)_w$ triplet and antitriplet superfields $\Sigma, \bar{\Sigma}$ with $U(1)_Y$ hypercharges 2 and -2 respectively (in this normalization, the hypercharge of the lepton doublet $l = (e^-, \nu)$ equals to -1). Σ and $\bar{\Sigma}$ are 2×2 matrices in $SU(2)_w$ space and their compositions are given by:

$$\Sigma_{ab} = \begin{pmatrix} \Sigma^{(++)} & \Sigma^{(+)} / \sqrt{2} \\ \Sigma^{(+)} / \sqrt{2} & \Sigma^{(0)} \end{pmatrix}_{ab}, \quad \bar{\Sigma}^{ab} = \begin{pmatrix} \bar{\Sigma}^{(-)} & \bar{\Sigma}^{(-)} / \sqrt{2} \\ \bar{\Sigma}^{(-)} / \sqrt{2} & \bar{\Sigma}^{(0)} \end{pmatrix}^{ab}, \quad (23)$$

where signs in superscripts indicate electric charges [e.g. $(++)$ stands with the double charged state].

With this field content, two scenarios which somewhat differ from each other, can be considered. We refer to them as type II-A soft and type II-B soft see-saw models.

Type II-A soft see-saw model

In this case, the R -charge assignment is given in Table 2 and relevant Kähler potential couplings are:

$$\mathcal{K}_m = \sum_f f^\dagger e^{g_a V_a} f, \quad \mathcal{K}_{nm} = \frac{\kappa}{M_{\text{Pl}}} X^\dagger \bar{\Sigma} \Sigma + \frac{X^\dagger X}{M^3} \left(\kappa_{A\bar{\Sigma}} \bar{\Sigma} h_u h_u + \frac{1}{2} \kappa_{A\Sigma} \Sigma l l \right) + \frac{\kappa_h}{M_{\text{Pl}}} X^\dagger h_u h_d + \text{h.c.} \quad (24)$$

The MSSM superpotential couplings will be same as given in (22). The couplings of Eq. (24) and SUSY breaking hidden sector, together with the μ and B -terms for the MSSM Higgs doublets, generate the following soft terms involving the scalar components of Σ and $\bar{\Sigma}$:

$$V(\Sigma, \bar{\Sigma}) = B_\Sigma \bar{\Sigma} \Sigma + A_\Sigma \bar{\Sigma} h_u h_u + \tilde{l}^T A_\Sigma \tilde{l} + \text{h.c.} + M_\Sigma^2 \Sigma^\dagger \Sigma + M_\Sigma^2 \bar{\Sigma}^\dagger \bar{\Sigma}. \quad (25)$$

Here Σ and $\bar{\Sigma}$ denote bosonic components. Their fermionic partners will be denoted by $\tilde{\Sigma}$ and $\tilde{\bar{\Sigma}}$ respectively. Integration of $\Sigma, \bar{\Sigma}$ states lead to the operator of Eq. (6) with approximate expression

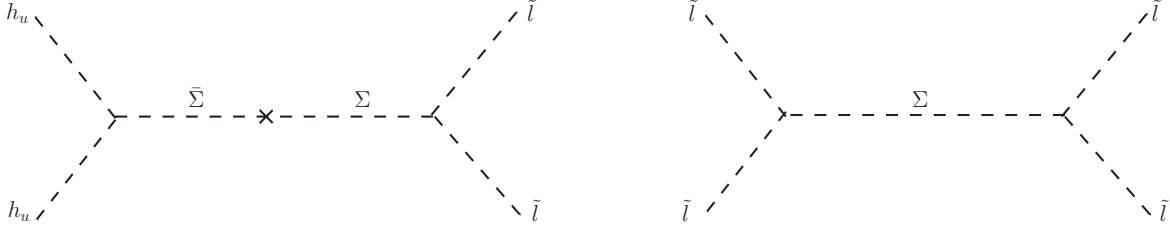


Figure 3: Diagrams generating $(\tilde{l}h_u)^2$ (left) and $(\tilde{l}\tilde{l})(\tilde{l}^*\tilde{l}^*)$ (right) terms by integration of scalar triplets $\Sigma, \bar{\Sigma}$.

for λ_4 given by:

$$\lambda_4 \simeq \frac{1}{M_\Sigma^2 M_{\bar{\Sigma}}^2} A_{\bar{\Sigma}} B_\Sigma^* A_\Sigma, \quad (26)$$

valid for $M_\Sigma M_{\bar{\Sigma}} \gtrsim |B_\Sigma|/3$. More accurate expression is given in Appendix B [see Eq. (B.10)]. The first diagram of Fig. 3 corresponds to the generation of this operator, which give loop induced neutrino masses as shown in Sect. 2.1. Also within this case, the needed suppression of λ_4 can be achieved, for example, by the selection $\frac{A_\Sigma}{M_\Sigma} \sim \frac{A_{\bar{\Sigma}}}{M_{\bar{\Sigma}}} \sim \frac{\sqrt{B_\Sigma}}{M_{\bar{\Sigma}}} \sim 10^{-2}$.

Via exchange of $\Sigma, \bar{\Sigma}$ states, the four slepton interaction operator is also generated, which in a same approximation (as Eq. (26)) is:

$$\mathcal{L}_{4\tilde{l}}^{\text{eff}} = \frac{1}{M_\Sigma^2} \left(\tilde{l}_a^T A_\Sigma \tilde{l}_b \right) \left(\tilde{l}_a^\dagger A_\Sigma^* \tilde{l}_b^* \right), \quad (27)$$

where $a, b = 1, 2$ are $SU(2)_w$ indices [see Eq. (B.11) for more accurate expression]. The corresponding diagram is the second one in Fig. 3. By 2-loop gaugino/higgsino dressing diagrams, the operator (27) will be converted to the four lepton operator. The latter in turn induce $e_i \rightarrow e_j e_m e_m$ rare decays, including processes such as $\tau \rightarrow 3\mu$, $\mu \rightarrow 3e$ etc. These will constrain the A_Σ matrix elements via experimental limits [13] and open window to probe this mechanism of neutrino mass generation.

Finally, the κ -term in Eq. (24) generates the mass term for the fermionic $\tilde{\Sigma}, \bar{\tilde{\Sigma}}$ components $\mu_\Sigma \tilde{\tilde{\Sigma}} \tilde{\Sigma}$ with $\mu_\Sigma \sim \kappa m_{3/2}$.

Type II-B soft see-saw model

In this modified version, the neutrino masses are induced at 2-loop level. The R -charges of the states are given in Table 3 and the Kähler potential couplings are

$$\mathcal{K}_m = \sum_f f^\dagger e^{g_a V_a} f, \quad \mathcal{K}_{nm} = \frac{\kappa}{M_{\text{Pl}}} X^\dagger \bar{\Sigma} \Sigma + \frac{X^\dagger X}{2M^3} \kappa_{A\Sigma} \Sigma l l + \frac{\kappa_h}{M_{\text{Pl}}} X^\dagger h_u h_d + \text{h.c.} \quad (28)$$

The $X^\dagger \bar{\Sigma} h_u h_u$ term is forbidden in the Kähler potential, but the following superpotential coupling, involving $\bar{\Sigma}$, is allowed:

$$W_{\bar{\Sigma}} = \lambda_{\bar{\Sigma}} \bar{\Sigma} h_u h_u. \quad (29)$$

The MSSM superpotential terms are same as given in Eq. (22). The coupling (29), in general

	X	h_u	Σ	$\bar{\Sigma}$	l
R	1	α	$1 + 2\alpha - \omega$	$\omega - 2\alpha$	$\frac{\omega}{2} - \frac{1}{2} - \alpha$
	h_d	e^c	q	u^c	d^c
R	$1 - \alpha$	$\frac{\omega}{2} + 2\alpha - \frac{1}{2}$	α_q	$\omega - \alpha_q - \alpha$	$\omega - \alpha_q + \alpha - 1$

Table 3: R charges in the Type II-B soft see-saw model, with $R_W = \omega$. By $\omega \neq 2m + 1$ ($m \in \mathbb{Z}$) the matter parity is automatic.

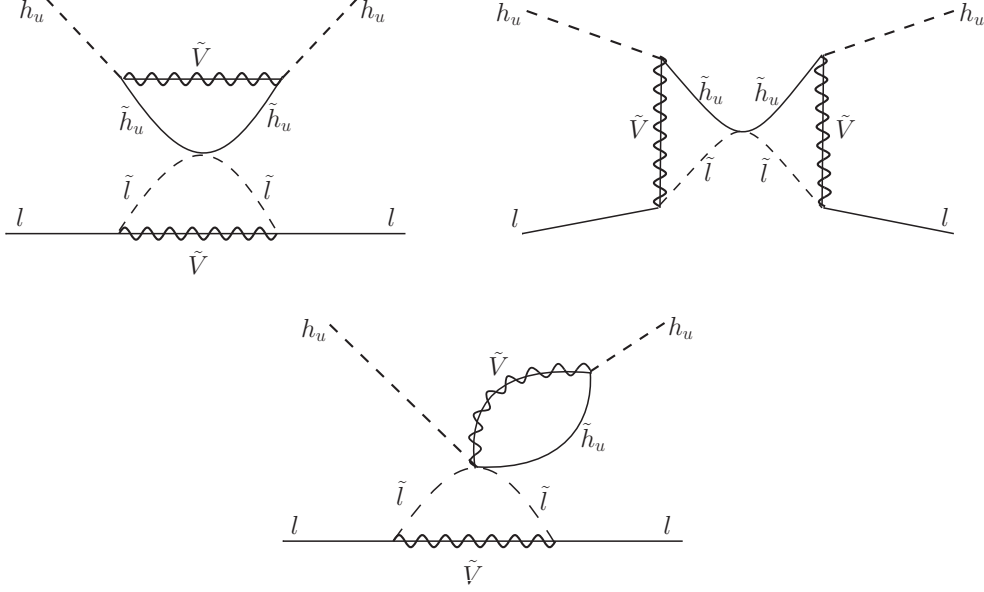


Figure 4: 2-loop diagrams responsible for neutrino masses within type II-B soft see-saw model.

would generate the trilinear soft term $A_{\bar{\Sigma}} \bar{\Sigma} h_u h_u$. However, within the SUSY breaking scenario we are considering, this A -term is suppressed:

$$A_{\bar{\Sigma}} \sim \lambda_{\bar{\Sigma}} m_{3/2} \frac{\langle X \rangle}{M_{Pl}} \lesssim 10^{-7} \lambda_{\bar{\Sigma}} m_{3/2} . \quad (30)$$

(see Eq. (A.25) and discussion therein).

Kähler potential terms of Eq. (28), besides higgsino μ and higgs B terms induce the potential terms:

$$B_{\Sigma} \bar{\Sigma} \Sigma + A_{\Sigma} \Sigma \tilde{l} \tilde{l} + \text{h.c} + M_{\Sigma}^2 \Sigma^{\dagger} \Sigma + M_{\bar{\Sigma}}^2 \bar{\Sigma}^{\dagger} \bar{\Sigma} . \quad (31)$$

With the couplings in (30), (31), integration of $\Sigma, \bar{\Sigma}$ states lead to the operator of Eq. (6) with expression for λ_4 having the same form as given in Eq. (26), but with extra strong suppression factor $\lesssim 10^{-7}$. Therefore, 1-loop contribution to the neutrino masses will be very suppressed and can be ignored. Neutrino masses will be induced at 2-loop level, which we discuss below.

The superpotential of Eq. (29) gives the following Yukawa interactions:

$$\int d^2\theta W_{\bar{\Sigma}} \rightarrow \lambda_{\bar{\Sigma}} \left(\bar{\Sigma} \tilde{h}_u \tilde{h}_u + 2 \tilde{\Sigma} \tilde{h}_u h_u \right). \quad (32)$$

These couplings, together with those given in (31), upon integration of scalar components from $\bar{\Sigma}, \Sigma$ superfields, induce the following $\Delta L = 2$ dimension 5 operator (λ'_5 -type of Eq. (5)):

$$\lambda'_5 (\tilde{l} \tilde{h}_u)^2, \quad \text{with} \quad \lambda'_5 \simeq \frac{1}{M_{\Sigma}^2 M_{\bar{\Sigma}}^2} \lambda_{\bar{\Sigma}} B_{\Sigma}^* A_{\Sigma}. \quad (33)$$

The way of derivation of this $d = 5$ operator is similar to that given in Appendix B. We just need to make replacement $A_{\bar{\Sigma}} h_u h_u \rightarrow \lambda_{\bar{\Sigma}} \tilde{h}_u \tilde{h}_u$ in Eq. (B.10). The operator (33), by the gaugino dressings at 2-loop give the $d = 5$ operator $\lambda_5 (l h_u)^2$ responsible for the neutrino mass. The relevant diagrams are given in Fig. 4. For consistency, one should also take into account the effective $d = 6$, $\Delta L = 2$ operators

$$-\frac{2}{M_{\Sigma}^2 \mu_{\Sigma}} \lambda_{\bar{\Sigma}} (\tilde{l}^T A_{\Sigma} \tilde{l}) (g \tilde{V} \tilde{h}_u h_u) + \text{h.c.} \quad (34)$$

(the λ'_6 -type coupling of Eq. (7)) which are induced by integration of the scalar and fermionic components from $\bar{\Sigma}, \Sigma$ superfields [by using the mass term $\mu_{\Sigma} \tilde{\Sigma} \tilde{\Sigma}$ and couplings (31), (32)]. In (34), $g \tilde{V} = \{g_1 \tilde{B}, g_2 \tilde{W}\}$ denote $U(1)_Y$ and/or $SU(2)_w$ gaugino contributions.

Care must be taken to treat properly divergences of the diagrams in Fig. 4. If the effective vortices (33) and (34) are used without specifying the model (the operators are induced from), the loops need to be cut off by the characteristic scale. Since within type II soft see-saw, these operators are obtained by integrating $\Sigma, \bar{\Sigma}$ states, full calculation can be done. With 'opening' vortices, the diagrams are shown in Fig. 5. It is obvious that log divergence of the first and third diagrams are coming from upper triangles corresponding to the parts of one loop renormalization of the soft $A_{\bar{\Sigma}}$ term (vanishing at tree level). Indeed,

$$\begin{aligned} \delta A_{\bar{\Sigma}} &= \delta A_{\bar{\Sigma}}^{(1)} + \delta A_{\bar{\Sigma}}^{(2)}, \quad \text{with} \\ \delta A_{\bar{\Sigma}}^{(1)} &= -\frac{\lambda_{\bar{\Sigma}}}{16\pi^2} \sum_{a=1}^3 c_a^{(1)} g_a^2 M_{\tilde{V}_a} \left(\ln \frac{\Lambda^2}{M_{\tilde{V}_a}^2} + \frac{\mu^2 (2M_{\tilde{V}_a}^2 - \mu^2)}{(M_{\tilde{V}_a}^2 - \mu^2)^2} \ln \frac{\mu^2}{M_{\tilde{V}_a}^2} - \frac{\mu^2}{\mu^2 - M_{\tilde{V}_a}^2} \right), \\ \delta A_{\bar{\Sigma}}^{(2)} &= -\frac{\lambda_{\bar{\Sigma}}}{16\pi^2} \sum_{a=1}^3 c_a^{(2)} g_a^2 M_{\tilde{V}_a} \left(\ln \frac{\Lambda^2}{M_{\tilde{V}_a}^2} - \frac{\mu^4 \ln \frac{\mu^2}{M_{\tilde{V}_a}^2}}{(\mu^2 - \mu_{\Sigma}^2)(\mu^2 - M_{\tilde{V}_a}^2)} - \frac{\mu_{\Sigma}^4 \ln \frac{\mu_{\Sigma}^2}{M_{\tilde{V}_a}^2}}{(\mu_{\Sigma}^2 - \mu^2)(\mu_{\Sigma}^2 - M_{\tilde{V}_a}^2)} \right), \\ \text{with} \quad c_a^{(1)} &= \left(\frac{6}{5}, 3, 0 \right), \quad c_a^{(2)} = \left(\frac{3}{5}, 6, 0 \right), \end{aligned} \quad (35)$$

where μ and μ_{Σ} are the MSSM higgsino μ -term and the mass of the fermionic $\tilde{\Sigma}, \tilde{\Sigma}$ states respectively, g_a is the gauge coupling (with $c_a^{(1,2)}$ being corresponding group theoretical factors and $M_{\tilde{V}_a}$ the corresponding gaugino's soft mass). In the last line of (35), $a = 1, 2, 3$ labels $U(1)_Y$, $SU(2)_w$ and $SU(3)_c$ gauge groups respectively (the coupling g_1 is taken in $SU(5)$ normalization). The $\ln \Lambda$ terms coincide to those obtained via integration of the RG equation for the $A_{\bar{\Sigma}}$ soft term. Expressions (35), after the renormalization, leave us with a non-divergent and finite parts. Assuming the

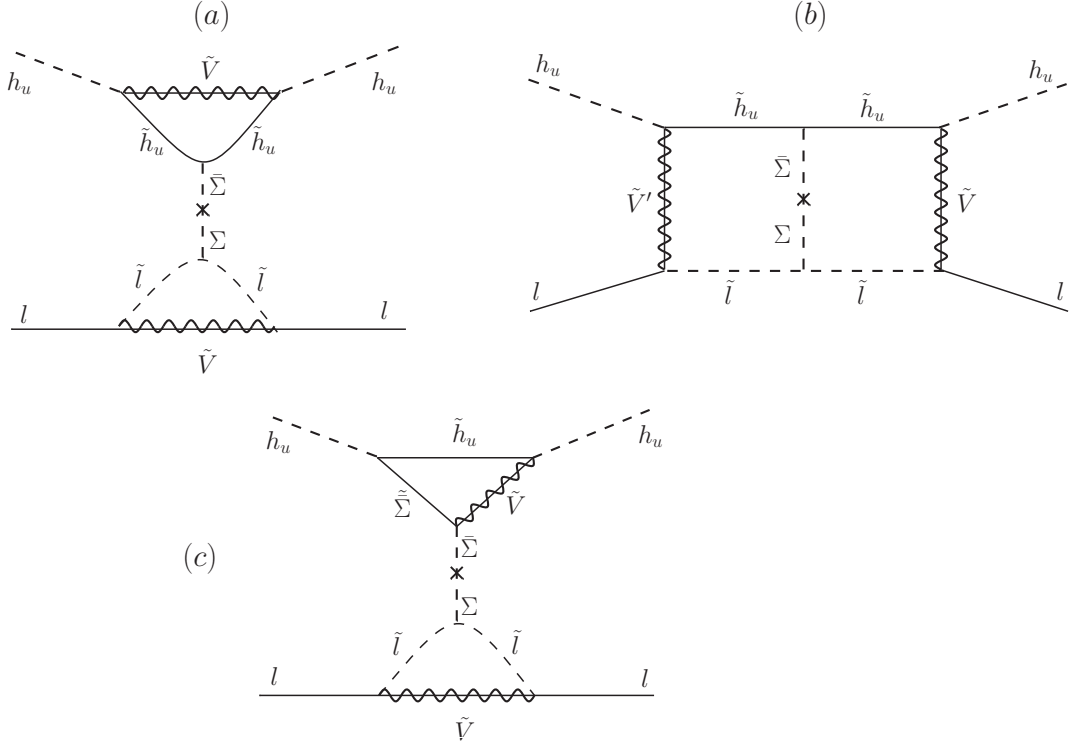


Figure 5: 2-loop diagrams responsible for neutrino masses within type II-B soft see-saw model (opening effective vortexes).

boundary condition $A_{\bar{\Sigma}}(M_{UV}) = 0$ at some scale M_{UV} (which can be the Planck mass, in case of gravity mediated SUSY breaking), at the SUSY scale M_S , we will have:

$$\text{at } \mu = M_S : \quad A_{\bar{\Sigma}}^{\text{eff}} = -\frac{\lambda_{\bar{\Sigma}}}{16\pi^2} \left(\sum_{a=1}^3 c_a g_a^2 M_{\tilde{V}_a} \right) \ln \frac{M_{UV}^2}{M_S^2}, \quad c_a = \left(\frac{9}{5}, 9, 0 \right). \quad (36)$$

Further, we compute the lower triangle diagrams of Fig. 5-(a),(c), which by the gaugino dressings convert $\tilde{l}\tilde{l}$ to ll . Then, integrate out the $\Sigma, \bar{\Sigma}$ states and obtain the contribution to the neutrino mass to be given by:⁷

$$(\delta M_{\nu}^{ij})_{\text{Fig.5-(a)+(c)}} = \frac{v_u^2 A_{\bar{\Sigma}}^{\text{eff}}}{16\pi^2} \frac{B_{\Sigma}^* A_{\Sigma}^{ij}}{M_{\Sigma}^2 M_{\Sigma}^2} \sum_{a=1}^3 \frac{c_a^{(1)} g_a^2}{M_{\tilde{V}_a}} f \left[\left(\frac{m_i}{M_{\tilde{V}_a}} \right)^2, \left(\frac{m_j}{M_{\tilde{V}_a}} \right)^2 \right], \quad (37)$$

where the function f is defined in Eq. (15) and m_i, m_j denote masses of \tilde{l}_i, \tilde{l}_j .

There is no any divergence from the diagram of Fig. 5-(b). Detailed evaluation of this diagram is given in Appendix C. Contribution to the neutrino mass matrix from the 2-loop diagram of Fig. 5-(b) is given by:

$$(\delta M_{\nu}^{ij})_{\text{Fig.5-(b)}} = \frac{v_u^2}{2} \mu^2 g^2 g'^2 A_{\Sigma}^{ij} B_{\Sigma}^* \lambda_{\bar{\Sigma}} J, \quad (38)$$

⁷Here we ignore the EW symmetry breaking effects, e.g. the gaugino-higgsino mixings.

where J is given in Eq. (C.11) and $\gamma_{\alpha nm}$ and other appearing factors are defined in Eqs. (C.9), (C.4) and (C.5) respectively.

Concluding, note that additional operators where h_u scalar is replaced by h_d^\dagger will be also induced. The relevance of this kind of operators will depend on the value of the VEV $\langle h_d^{(0)} \rangle$. The latter, on the other hand, depends on the parameter $\tan \beta$. Thus, the details and numerical results will depend on SUSY spectroscopy, which should be taken such that LHC constraints are satisfied. The latter study is beyond the scope of this work.

3.3 Type III Soft See-Saw

In this case, the MSSM is extended with $SU(2)_w$ triplets superfields T with zero hypercharges. For a realistic neutrino sector, at least two such superfields should be introduced. The matrix representation of this superfield(s) has the form

$$T_b^a = \begin{pmatrix} T^{(0)}/\sqrt{2} & T^{(-)} \\ T^{(+)} & -T^{(0)}/\sqrt{2} \end{pmatrix}_b^a, \quad (39)$$

with superscripts indicating the electric charges of the fragments. In this case, the R charges are:

$$\begin{aligned} R_W = \omega, \quad R_X = 1, \quad R_{h_u} = \alpha, \quad R_T = \frac{1}{2}, \quad R_l = -\frac{1}{2} - \alpha, \quad R_{h_d} = 1 - \alpha, \\ R_{e^c} = \omega + 2\alpha - \frac{1}{2}, \quad R_q = \alpha_q, \quad R_{u^c} = \omega - \alpha - \alpha_q, \quad R_{d^c} = \omega - \alpha_q + \alpha - 1. \end{aligned} \quad (40)$$

Note that with $\omega \neq m + \frac{1}{2}$ ($m \in \mathbb{Z}$) the matter parity is automatic. The superpotential couplings with the T states will be forbidden, while the Kähler potential couplings will be

$$\mathcal{K}_m = \sum_f f^\dagger e^{g_a V_a} f, \quad \mathcal{K}_{nm} = \frac{\kappa}{2M_{\text{Pl}}} X^\dagger T T + \frac{X^\dagger X}{M^3} \kappa_A l T h_u + \frac{\kappa_h}{M_{\text{Pl}}} X^\dagger h_u h_d + \text{h.c.} \quad (41)$$

The R charge selection of Eq. (40) is consistent with MSSM superpotential couplings given in Eq. (22). From the coupling of (41), likewise of previous cases, together with the MSSM μ and B -terms, the following soft SUSY breaking terms are induced:

$$\frac{1}{2} T^T B_T T + \tilde{l}^T A_T T h_u + \text{h.c.} + T^\dagger M_T^2 T, \quad (42)$$

where by T the scalar components are denoted. Their fermionic partners \tilde{T} gain the mass $\frac{1}{2}\mu_T \tilde{T} \tilde{T}$ with $\mu_T \sim \kappa m_{3/2}$. (Once more, we refer the reader to Appendix A for details.) Now, one can easily verify that integration of T 's scalar components, via (42) couplings induce the $\Delta L = 2$ operator of Eq. (6) with:

$$\lambda_4 \simeq \frac{1}{2} A_T \frac{1}{M_T^2} B_T^\dagger \frac{1}{M_T^2} A_T^T. \quad (43)$$

This expression is approximate and works well with $M_T^2 \gtrsim B_T/3$. The derivation of this expression is very similar to that corresponding to the case of Type I soft see-saw model, presented in Appendix B.⁸ With the λ_4 coupling the neutrino mass will be generated (as discussed in Sect. 2.1) at

⁸For obtaining (43) one can do the replacements $B_N \rightarrow B_T$ and $A_N \rightarrow A_T$ in Eq. (B.7) and then use the proper approximation.

1-loop level via diagram shown in Fig. 1. 1-loop contribution will dominate if $Y_T \tilde{l} \tilde{T} h_u$ type Yukawa interactions, induced through the Kähler potential (with $Y_T \sim \langle X \rangle / M_{Pl}$) will be adequately suppressed. This requires $\langle X \rangle \lesssim 2 \cdot 10^{11}$ GeV. For this case [unlike the Eq. (21)] we will not have low bound on $\langle X \rangle$, because the states from T can decay via MSSM gauge interactions.

From the potential (42), upon integration of the T states, between \tilde{l} and T , the mixing of the order $\sim A_T \frac{v_u}{M_T^2}$ is emerged. This, in turn, via T -gaugino (higgsino) loops will induce $e_i \rightarrow e_j \gamma$ decays. However, with $M_T \sim \text{few} \cdot \text{TeV}$ and $A_T/M_T \lesssim 1/10$, the $\tilde{l} - T$ mixing will be $\lesssim 10^{-2}$. This will be enough for adequate suppression of such rare processes. Much will also depend on a details of the flavor structure of A_T and M_T^2 matrices, which require separate investigation and is not pursued here.

Concluding, let us emphasize that the decays of the states from $\Sigma, \bar{\Sigma}$ and T (within type II and type III soft see-saw scenarios respectively) in SM and LSP states proceed via EW interactions and there is no low bound on the value of $\langle X \rangle$ (unlike to the type I soft see-saw scenario. See the discussion before and after Eq. (20)).

4 Discussion and Outlook

In this paper we have presented possibilities for radiative neutrino mass generation within SUSY theories. Suggested mechanisms open broad prospects for further investigations.

Since extensions, we have suggested, have the lepton number violation in the soft SUSY breaking sector with new states near the TeV scale, the models would have peculiar collider signatures. This gives possibilities for testing the origin of neutrino masses at accelerator experiments. This issue deserves separate investigation in a spirit of Refs. [14], [15]. It would be also interesting to exercise the lepton number violating higher dimensional operators (such as $d \geq 7$, $\Delta L = 2$ couplings studied in different scenarios earlier [16], [15]) and investigate how they may emerge from the soft SUSY breaking terms.

Since within presented models new states lie near the TeV scale and they have some family dependent interactions, one expects to have new contributions to the rare processes such as $\mu \rightarrow e \gamma$, $\tau \rightarrow (e, \mu) \gamma$, $\mu \rightarrow 3e$, etc. These also could be the signatures of the presented scenarios and may serve for models' test. Similar concern to new possible contributions to the EW precision parameters (T, S and U).

Finally, would be challenging to embed considered models in SUSY Grand Unification (GUT) such as $SU(5)$ and $SO(10)$ GUTs. Because of the GUT symmetry, new relations and constraints would emerge, making models more predictive. These and related issues will be addressed elsewhere.

Acknowledgments

We thank K.S. Babu, B. Bajc and M. Nemevsek for discussions and helpful comments. The work is partially supported by Shota Rustaveli National Science Foundation (Contracts No. 31/89 and No. DI/12/6-200/13). Z.T. thanks CETUP* (Center for Theoretical Underground Physics and Related Areas) for its hospitality and partial support during 2015 and 2016 Summer Programs. Z.T. also would like to express a special thanks to the Mainz Institute for Theoretical Physics (MITP) for its hospitality and support.

A SUSY Breaking. μ , B and A -type Couplings

In the superconformal formulation, the supergravity Lagrangian density is given by [17], [18]

$$\mathcal{L}_D + \mathcal{L}_F \quad \text{with}$$

$$\mathcal{L}_D = -3 \int d^4\theta e^{-\mathcal{K}/(3M_{Pl}^2)} \phi^+ \phi, \quad \mathcal{L}_F = \frac{1}{M_{Pl}^3} \int d^2\theta \phi^3 W + \frac{1}{4} \int d^2\theta f_{IJ} \mathcal{W}^{\alpha I} \mathcal{W}_\alpha^J + \text{h.c.} \quad (\text{A.1})$$

where \mathcal{K} and $W(\Phi)$ are the Kähler potential and the superpotential respectively, while $f_{IJ}(\Phi)$ is the gauge kinetic function (for the chiral superfield strength $\mathcal{W}^{\alpha I}$ obtained from the vector superfield V^I). ϕ is the compensator chiral superfield. Superspace integrals of (A.1) (and throughout this paper) should be understood as F and D-densities of (conformal) SUGRA as given in refs. [17], [18].

In general, the scalar potential

$$V = V_F + V_D, \quad (\text{A.2})$$

consists of two parts. The D -term potential

$$V_D = \frac{1}{2} \sum_a g_a^2 D_a^2, \quad (\text{A.3})$$

and the F -term potential V_F . The latter is obtained from (A.1) by integrating F -terms. Doing so, setting $\phi = M_{Pl}$ and going to the Einstein-frame one obtain:

$$V_F = e^{\mathcal{K}/M_{Pl}^2} \left(D_J \bar{W} \mathcal{K}^{\bar{J}I} D_I W - \frac{3}{M_{Pl}^2} |W|^2 \right), \quad \text{with} \quad D_I \equiv \partial_I + \frac{1}{M_{Pl}^2} \mathcal{K}_I. \quad (\text{A.4})$$

Few comments about the definitions are in order. In the 'covariant' derivative D_I (with respect to the field Φ_I) we have: $\partial_I \equiv \frac{\partial}{\partial \Phi_I}$ and $\mathcal{K}_I \equiv \frac{\partial \mathcal{K}}{\partial \Phi_I}$. The object $\mathcal{K}^{\bar{J}I}$ is an inverse of the matrix $\mathcal{K}_{I\bar{J}}$ build from the second derivatives of the Kähler potential $\mathcal{K}_{I\bar{J}} = \frac{\partial^2 \mathcal{K}}{\partial \Phi_I \partial \Phi_{\bar{J}}^\dagger}$. Thus,

$$\mathcal{K}_{I\bar{M}} \mathcal{K}^{\bar{M}J} = \delta_I^J, \quad \mathcal{K}^{\bar{I}M} \mathcal{K}_{M\bar{J}} = \delta_{\bar{J}}^{\bar{I}}.$$

The DW can be defined as a column, while $D\bar{W}$ is its hermitian conjugate row:

$$D\bar{W} = ((D_1 W)^*, (D_2 W)^*, \dots).$$

Thus, the first entry in the brackets of Eq. (A.4) can be written as $D\bar{W}(\mathcal{K}'')^{-1}DW$.

We will be dealing with superfields X_i of the visible sector and with superfield X through which the SUSY breaking takes place in a hidden sector. For all chiral superfields unified notation $\Phi_I = \{X, X_i\}$ is used. The Kähler potential \mathcal{K} , together with minimal quadratic (kinetic) terms, will also include non-holomorphic higher order terms:

$$\mathcal{K} = \sum_I \Phi_I^\dagger e^{g_a V_a} \Phi_I + \Phi_I^\dagger (\kappa_{(2)}^{IJK} \Phi_J \Phi_K + \kappa_{(3)}^{IJKM} \Phi_J \Phi_K \Phi_M + \dots) + \dots + \text{h.c.} \quad (\text{A.5})$$

The superpotential

$$W = W_v(X_i) + W_h(X) \quad (\text{A.6})$$

is the sum of the visible and hidden sector superpotentials, denoted by $W_v(X_i)$ and $W_h(X)$ respectively. The form of $W_h(X)$ should insure SUSY breaking, i.e. $F_X \neq 0$. Within our consideration, the couplings in Eq. (A.5), in combination with the superpotential, will be responsible for generation of μ type terms as well as for B and A -type soft SUSY breaking terms. Namely, with the expression of the scalar potential we can calculate the mass² terms (corresponding to soft mass² and B -terms) and trilinear scalar couplings (corresponding to the soft SUSY breaking A -terms):

$$m_{ij}^2 = \frac{\partial^2 V}{\partial X_i \partial X_j^\dagger}, \quad B_{ij} = \frac{\partial^2 V}{\partial X_i \partial X_j}, \quad A_{ijk} = \frac{\partial^3 V}{\partial X_i \partial X_j \partial X_k}. \quad (\text{A.7})$$

More detailed discussion about these couplings will be given in the next subsection after discussing the details of the SUSY breaking.

A.1 SUSY Breaking via Polonyi Superpotential

For the SUSY breaking hidden sector superpotential $W_h(X)$ we consider Polonyi superpotential [19] of the form:

$$W_h = JX + C. \quad (\text{A.8})$$

The coupling J insures non zero F_X , while the constant C is needed to cancel cosmological constant. The hidden sector superpotential (as usually) explicitly breaks the R -symmetry.⁹ This may be considered as a soft breaking, because it does not makes troubles in a visible sector. With X 's minimal Kähler potential $\mathcal{K} = X^\dagger X$, the superpotential (A.8), by proper selection of J and C gives SUSY breaking Minkowski vacuum, but with $\langle X \rangle \sim M_{Pl}$. The reason for this is the following. Minimum condition in a Minkowski vacuum fixes J and C . With this, the potential for X has the linear term $\sim m_{3/2}^2 M_{Pl} X$ and the quadratic term $\sim m_{3/2}^2 |X|^2$ (i.e. potential's curvature) and the VEV of X is basically set from the ratio of these two terms: $\langle X \rangle \sim m_{3/2}^2 M_{Pl} / m_{3/2}^2 = M_{Pl}$.

The situation will change, i.e. one can get desirable value of F_X and suppressed $\langle X \rangle$, with higher order terms in the Kähler potential. For our purposes will be enough to include the coupling $\Delta \mathcal{K}(X) = -(X^\dagger X)^2 / (4M_*^2)$. As we will see, with $M_* \ll M_{Pl}$ the scalar potential will develop high curvature ($\sim m_{3/2}^2 \frac{M_{Pl}^2}{M_*^2}$), which will suppress the VEV of X . Thus, we will consider X 's Kähler potential:

$$\mathcal{K}(X) = X^\dagger X - \frac{1}{4M_*^2} (X^\dagger X)^2 \quad (\text{A.9})$$

and analyze this case in details. Since the gravitino mass is given by [11], [21]:

$$m_{3/2} = e^{\langle \mathcal{K} \rangle / (2M_{Pl}^2)} \frac{\langle W \rangle}{M_{Pl}^2}, \quad (\text{A.10})$$

and we are looking for a solution with suppressed $\langle X \rangle$, we will use the parametrization

$$J = \bar{m} M_{Pl}, \quad C = c \bar{m} M_{Pl}^2, \quad \alpha = M_{Pl} / M_*, \quad x = X / M_{Pl}, \quad (\text{A.11})$$

⁹This is desirable to avoid cosmological difficulties with massless R -axion, emerging from the spontaneous breaking of the continuous R -symmetry. For interesting interconnection between R -symmetry and SUSY breaking see Ref. [20].

where \bar{m} will turn out to be of the order of $m_{3/2}$ and c is a constant. With these, using in (A.4) forms of (A.8) and (A.9), we will get the potential:

$$V = \bar{m}^2 M_{Pl}^2 e^K \left(\frac{1}{1 - \alpha^2 |x|^2} \left| 1 + x^* \left(1 - \frac{\alpha^2}{2} |x|^2 \right) (x + c) \right|^2 - 3 |x + c|^2 \right) \quad (\text{A.12})$$

One can easily check that in the region $|x| < 1/\alpha$, with $\alpha \gg 1$ (i.e. for $M_* \ll M_{Pl}$), the potential has the minimum. The constant c can be selected in such a way that the cosmological constant is zero in this minimum. Doing this, upon the minimization of (A.12), we find:

$$\langle X \rangle = M_{Pl} \langle x \rangle \simeq \frac{2}{\sqrt{3}} \frac{M_*^2}{M_{Pl}} \left(1 - \frac{11}{3} \left(\frac{M_*}{M_{Pl}} \right)^2 + \frac{74}{3} \left(\frac{M_*}{M_{Pl}} \right)^4 + \dots \right), \quad (\text{A.13})$$

$$c \simeq \frac{1}{\sqrt{3}} \left(1 - \frac{2}{3} \left(\frac{M_*}{M_{Pl}} \right)^2 + \frac{14}{9} \left(\frac{M_*}{M_{Pl}} \right)^4 - \frac{190}{27} \left(\frac{M_*}{M_{Pl}} \right)^6 + \dots \right). \quad (\text{A.14})$$

In finding these we have used an expansion with powers of M_*/M_{Pl} . So, the solution can be found up to the needed accuracy keeping appropriate power of M_*/M_{Pl} . With (A.13), (A.14) and (A.11), from Eq. (A.10) we find:

$$\bar{m} \simeq \sqrt{3} m_{3/2}. \quad (\text{A.15})$$

In the minimum, the masses of real and imaginary scalar components of the X field will be $\simeq \sqrt{3} m_{3/2} M_{Pl} / M_*$. The region $|x| < 1/\alpha$ has minimum with a SUSY breaking. Although, the VEV of the X field can be strongly suppressed, the F_X -term and $D_X W$ in the minimum are of the order of $\sim m_{3/2} M_{Pl}$:

$$\langle F_X \rangle \simeq \sqrt{3} m_{3/2} M_{Pl}, \quad \langle D_X W \rangle = \langle \partial_X W + \frac{\mathcal{K}_X}{M_{Pl}^2} W \rangle \simeq \sqrt{3} m_{3/2} M_{Pl}. \quad (\text{A.16})$$

In the regions $|x| > 1/\alpha$ the potential is not bounded from below. However, the tunneling from the branch of $|x| < 1/\alpha$ to the either other branches are extremely suppressed. As far as the Kähler coupling $-(X^\dagger X)^2 / (4M_*^2)$ [the second term of Eq. (A.9)] is concerned, it can be generated by integration of states of mass $\sim M_*$ which couple with the X field. For instance, having two chiral superfields $\Omega, \bar{\Omega}$ and the superpotential couplings $\lambda X \Omega^2 + M_\Omega \Omega \bar{\Omega}$, at 1-loop level the Kähler potential receives the quartic correction¹⁰ $\Delta \mathcal{K}(X) = -\frac{|\lambda|^4}{12\pi^2 M_\Omega^2} (X^\dagger X)^2$. This justifies analysis performed above.

Within considered framework, the visible sector superfields X_i will couple with X via higher order terms in the Kähler potential. Within our consideration, the latter will have the form:

$$\mathcal{K}(X_i) = X_i^\dagger X_i + \frac{X_i^\dagger X}{6\bar{M}^3} \kappa_A^{ijk} X_i X_j X_k + \frac{X^\dagger}{2M_{Pl}} \kappa^{ij} X_i X_j + \text{h.c.} \quad (\text{A.17})$$

These couplings, as shown in [12] will generate μ , B and A -type terms, as well as the Yukawa couplings. From the expression of the F -term potential (A.4), we have following relevant terms:

$$-\frac{|F_X|^2}{6\bar{M}^3} \kappa_A^{ijk} X_i X_j X_k - \frac{F_X W^*}{2M_{Pl}^3} \kappa^{ij} X_i X_j + \text{h.c.} = -\frac{m_{3/2}^2 M_{Pl}^2}{2\bar{M}^3} \kappa_A^{ijk} X_i X_j X_k - \frac{\sqrt{3}}{2} m_{3/2}^2 \kappa^{ij} X_i X_j + \text{h.c.} \quad (\text{A.18})$$

¹⁰Simple way for computing the loop corrections to the Kähler potential is to use formalism given in Ref. [22].

Therefore we have obtained the B -terms:

$$B^{ij} = -\sqrt{3}m_{3/2}^2\kappa^{ij}, \quad (\text{A.19})$$

of the needed value. The A -terms will have desirable values with $\bar{M} = (3m_{3/2}M_{Pl}^2)^{1/3} \sim 10^{-5}M_{Pl}$. With this scale, we will have

$$A^{ijk} = -m_{3/2}\kappa_A^{ijk}, \quad \text{with} \quad \bar{M} = (3m_{3/2}M_{Pl}^2)^{1/3}. \quad (\text{A.20})$$

For the fermionic states Ψ_i (coming from X_i superfield) the μ -terms $\frac{1}{2}\mu^{ij}\Psi_i\Psi_j$ can be calculated from the mass formulae [21]:

$$M_{ij} = \frac{m_{3/2}M_{Pl}^2}{\langle W \rangle} \left((\partial_{X_i} + \frac{\mathcal{K}_{X_i}}{M_{Pl}^2})\nabla_{X_j}W - \Gamma_{X_i X_j}^I \nabla_I W \right) - \frac{3}{2} \frac{M_{Pl}^2}{\langle W \rangle^2} (\nabla_{X_i}W)(\nabla_{X_j}W). \quad (\text{A.21})$$

Using this and the forms of (A.8), (A.9) and (A.17), we get:

$$\mu^{ij} = -\sqrt{3}m_{3/2}\kappa^{ij}. \quad (\text{A.22})$$

From the mass formulae (A.21), we can also extract The Yukawa couplings. With the definition of the Yukawa coupling $\frac{1}{2}Y^{ijk}X_i\Psi_j\Psi_k$, with $\nabla_{X_i}W = \partial_{X_i}W$ (valid within this model) and using (A.17), (A.20), we obtain

$$Y^{ijk} = \frac{2A^{ijk}}{\sqrt{3}m_{3/2}} \frac{\langle X \rangle}{M_{Pl}} = -\frac{2}{\sqrt{3}}\kappa_A^{ijk} \frac{\langle X \rangle}{M_{Pl}} \simeq -\frac{4}{3} \left(\frac{M_*}{M_{Pl}} \right)^2 \kappa_A^{ijk}. \quad (\text{A.23})$$

Since, $\langle X \rangle$ can be strongly suppressed, the Yukawa coupling also can have desirable suppression. Namely, with $\frac{M_*}{M_{Pl}} \lesssim 3 \cdot 10^{-4}$, if the Y^{ijk} corresponds to the neutrino Dirac Yukawa couplings Y_ν , then according to Eq. (A.23) they will be $|Y_\nu| \lesssim 10^{-7}$, i.e. irrelevant for the neutrino masses.

If the (visible) superpotential involves the trilinear Yukawa-type terms

$$W^{(3)} = \frac{1}{6}\lambda^{ijk}X_iX_jX_k, \quad (\text{A.24})$$

then, using (A.4) and (A.7) we can see that the corresponding A -term is also induced due to λ^{ijk} coupling, but is strongly suppressed by the M_*^2/M_{Pl}^2 factor:

$$A_\lambda^{ijk} \simeq \frac{1}{3} \left(\frac{M_*}{M_{Pl}} \right)^2 m_{3/2}\lambda^{ijk}. \quad (\text{A.25})$$

We close this appendix by commenting on the possibility of the soft gaugino mass generation. Since within our framework we are applying the R -symmetry, the corresponding operator should be consistent with it. For instance, if the superpotential's R charge is selected as $\omega = 4$,¹¹ then we also have $R(\mathcal{W}^\alpha\mathcal{W}_\alpha) = \omega = 4$ and the D -term effective operator responsible for gaugino mass will be $\int d^4\theta \left(\frac{X^\dagger}{X} \right)^2 \frac{\mathcal{W}^\alpha\mathcal{W}_\alpha}{M_{Pl}}$. From this, the gaugino mass will be $M_{\tilde{V}} \sim \frac{1}{M_{Pl}} \left(\frac{X^\dagger}{X} \right)_D^2 = -\frac{2|F_X|^2}{M_{Pl}\langle X \rangle^2} \simeq -m_{3/2} \frac{27m_{3/2}}{8M_{Pl}(M_*/M_{Pl})^4}$, which for $M_*/M_{Pl} \sim 2 \cdot 10^{-4}$, $m_{3/2} \sim \text{TeV}$, gives the desirable value $M_{\tilde{V}} \sim m_{3/2}$. Different possibility, for geugino mass($\sim m_{3/2}$) generation would be to include the operator $\int d^2\theta \frac{X}{M_{Pl}} \mathcal{W}^\alpha\mathcal{W}_\alpha$. The latter, although explicitly breaks R -symmetry (similar to hidden sector superpotential), but do not spoil any phenomenology, also can be considered as a plausible option.

¹¹With this selection no phenomenologically harmful coupling will be allowed within models we are considering [see Tables 1, 2, 3 and Eq. (40)].

B Deriving Effective Couplings

λ_4 Coupling induced by \tilde{N} states

Here we derive the effective operator (6) obtained by integrating out the states \tilde{N} . The relevant terms are given in the potential of Eq. (18). Not taking into account the EW symmetry breaking effects, we set D -terms to zero and do not consider their effects.

Since we are deriving an effective operator, relevant at low energies, we ignore the kinetic terms (setting momenta to zero). Then the equations of motion for \tilde{N}, \tilde{N}^* are:

$$\begin{aligned} -\frac{\partial \mathcal{L}(\tilde{N})}{\partial \tilde{N}_p^*} &= \frac{\partial V(\tilde{N})}{\partial \tilde{N}_p^*} = (M_{\tilde{N}}^2)^{pj} \tilde{N}_j + A_N^{*\alpha p} \tilde{l}_\alpha^* h_u^* + B_N^{*pj} \tilde{N}_j^* = 0 \\ -\frac{\partial \mathcal{L}(\tilde{N})}{\partial \tilde{N}_p} &= \frac{\partial V(\tilde{N})}{\partial \tilde{N}_p} = (M_{\tilde{N}}^2)^{jp} \tilde{N}_j^* + A_N^{\alpha p} \tilde{l}_\alpha h_u + B_{pj} \tilde{N}_j = 0, \end{aligned} \quad (\text{B.1})$$

where p, j indices numerate RHN states, while $\alpha (= 1, 2, 3)$ is a family index. We will be interested in the case when the mass scales $M_{\tilde{N}}^2$ are larger than the terms $A_N \langle h_u^{(0)} \rangle$. Then the mixings between \tilde{N} and $\tilde{\nu}$ can be ignored at the leading order. Therefore, \tilde{N} states can be integrated from Eqs. (B.1). The latter in a matrix form are:

$$\begin{pmatrix} M_{\tilde{N}}^2 & B_N^\dagger \\ B_N & (M_{\tilde{N}}^2)^T \end{pmatrix} \begin{pmatrix} \tilde{N} \\ \tilde{N}^* \end{pmatrix} = \begin{pmatrix} -A_N^\dagger \tilde{l}^* h_u^* \\ -A_N^T \tilde{l} h_u \end{pmatrix}, \quad (\text{B.2})$$

where the matrix at l.h.s of (B.2) contains the block sub-matrices. Having in general the block matrix of the form:

$$\mathcal{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad (\text{B.3})$$

it's inverse (in case $\text{Det} A \neq 0$ and $\text{Det} D \neq 0$) is given by [11]:

$$\mathcal{M}^{-1} = \begin{pmatrix} (A - BD^{-1}C)^{-1} & -A^{-1}B(D - CA^{-1}B)^{-1} \\ -D^{-1}C(A - BD^{-1}C)^{-1} & (D - CA^{-1}B)^{-1} \end{pmatrix}. \quad (\text{B.4})$$

Using this, from (B.2) we obtain

$$\tilde{N} = -\left(M_{\tilde{N}}^2 - B_N^\dagger ((M_{\tilde{N}}^2)^T)^{-1} B_N\right)^{-1} A_N^\dagger \tilde{l}^* h_u^* + (M_{\tilde{N}}^2)^{-1} B_N^\dagger \left((M_{\tilde{N}}^2)^T - B_N (M_{\tilde{N}}^2)^{-1} B_N^\dagger\right)^{-1} A_N^T \tilde{l} h_u \quad (\text{B.5})$$

Plugging the solution (B.5) back in (18), after grouping various terms and some simplifications, we obtain the effective $d = 4$, $\Delta L = 2$ interaction term

$$\mathcal{L}_{\text{eff}}^{\Delta L=2} = \tilde{l}^T \lambda_4 \tilde{l} h_u h_u + \text{h.c.} \quad (\text{B.6})$$

i.e. that given in Eq. (6), with:

$$\lambda_4 = \frac{1}{2} A_N (M_{\tilde{N}}^2)^{-1} B_N^\dagger \left((M_{\tilde{N}}^2)^T - B_N (M_{\tilde{N}}^2)^{-1} B_N^\dagger\right)^{-1} A_N^T. \quad (\text{B.7})$$

The corresponding diagram is given in Fig. 2. In the case of relatively small B_N -terms (e.g. $M_{\tilde{N}}^2 \gtrsim B_N/3$), the (B.7) is simplified to the form given in Eq. (19).

λ_4 Coupling and $\mathcal{L}_{\text{eff}}^{4\tilde{l}}$ induced by Σ and $\bar{\Sigma}$ states

In this case, we integrate out the scalar components from the superfields Σ and $\bar{\Sigma}$. The latter will be denoted by same symbols as superfields they are coming from. The relevant potential terms are given in Eq. (25). As for the case with \tilde{N} states, here we also ignore kinetic terms, EW symmetry breaking effects and also mixings of components of h_u, h_d with corresponding states of $\Sigma, \bar{\Sigma}$. Eqs. of motion for Σ^* and $\bar{\Sigma}$ (only two of them are independent) are:

$$\begin{aligned} -\frac{\partial \mathcal{L}}{\partial \Sigma^*} &= \frac{\partial V(\Sigma, \bar{\Sigma})}{\partial \Sigma^*} = M_{\Sigma}^2 \Sigma + B_{\Sigma}^* \bar{\Sigma}^* + \tilde{l}^\dagger A_{\Sigma}^* \tilde{l}^* = 0 \\ -\frac{\partial \mathcal{L}}{\partial \bar{\Sigma}} &= \frac{\partial V(\Sigma, \bar{\Sigma})}{\partial \bar{\Sigma}} = M_{\bar{\Sigma}}^2 \bar{\Sigma}^* + B_{\Sigma} \Sigma + A_{\Sigma} h_u h_u = 0. \end{aligned} \quad (\text{B.8})$$

From (B.8) we find solutions:

$$\Sigma = \frac{1}{M_{\Sigma}^2 M_{\bar{\Sigma}}^2 - |B_{\Sigma}|^2} \left(A_{\Sigma} B_{\Sigma}^* h_u h_u - M_{\bar{\Sigma}}^2 (\tilde{l}^\dagger A_{\Sigma}^* \tilde{l}^*) \right), \quad \bar{\Sigma}^* = \frac{1}{M_{\Sigma}^2 M_{\bar{\Sigma}}^2 - |B_{\Sigma}|^2} \left(M_{\Sigma}^2 A_{\Sigma} h_u h_u + B_{\Sigma} (\tilde{l}^\dagger A_{\Sigma}^* \tilde{l}^*) \right). \quad (\text{B.9})$$

Plugging these solutions back into the potential (25), we get λ_4 -type [of Eq. (6)] $d = 4$, $\Delta L = 2$ interaction term

$$\mathcal{L}_{\text{eff}}^{\Delta L=2} = -\frac{A_{\Sigma} B_{\Sigma}^*}{M_{\Sigma}^2 M_{\bar{\Sigma}}^2 - |B_{\Sigma}|^2} (\tilde{l}^T A_{\Sigma} \tilde{l}) h_u h_u + \text{h.c} \quad (\text{B.10})$$

and also quartic term with respect to \tilde{l} :

$$\mathcal{L}_{\text{eff}}^{4\tilde{l}} = \frac{M_{\Sigma}^2}{M_{\Sigma}^2 M_{\bar{\Sigma}}^2 - |B_{\Sigma}|^2} (\tilde{l}^T A_{\Sigma} \tilde{l}) (\tilde{l}^\dagger A_{\Sigma}^* \tilde{l}^*). \quad (\text{B.11})$$

With scales $M_{\Sigma} M_{\bar{\Sigma}} \gtrsim |B_{\Sigma}|/3$, the couplings of (B.10) and (B.11) reduce to those given in Eqs. (26) and (27) respectively.

C Evaluating 2-loop Diagram of Fig. 5-(b)

The amplitude, corresponding to the diagram of Fig. 5-(b), is given by

$$\mathcal{M} = 2 \cdot 2 \frac{g^2 g'^2}{4} A_{\Sigma}^{ij} B_{\Sigma}^* \lambda_{\bar{\Sigma}} \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{P_L (\not{p} + M_{\tilde{V}}) \mathbf{C} P_R (\not{p} + \mu) P_R \mathbf{C} (\not{q} + \mu) P_R (\not{q} + M_{\tilde{V}'}) \mathbf{C} P_L}{(p^2 - M_{\tilde{V}}^2)(p^2 - \mu^2)(q^2 - \mu^2)(q^2 - M_{\tilde{V}'}^2)(q^2 - m_i^2)(p^2 - m_j^2)} \frac{1}{[(p+q)^2 - M_{\Sigma}^2][(p+q)^2 - M_{\bar{\Sigma}}^2]} \quad (\text{C.1})$$

where \mathbf{C} and $P_{L,R}$ are charge conjugation and projection matrices respectively. Using the properties

$$\not{p} P_{L,R} = P_{R,L} \not{p}, \quad \mathbf{C} P_{L,R} = P_{L,R} \mathbf{C}, \quad \mathbf{C}^{-1} = -\mathbf{C}, \quad P_{R,L}^2 = P_{R,L}, \quad P_R P_L = 0 = P_L P_R \quad (\text{C.2})$$

the numerator of the integral in (C.1) gets simplified to $\mu^2(pq) P_L \mathbf{C}^{-1}$ and we obtain:

$$\mathcal{M} = \mu^2 g^2 g'^2 A_{\Sigma}^{ij} B_{\Sigma}^* \lambda_{\bar{\Sigma}} P_L \mathbf{C}^{-1} J, \quad \text{with}$$

$$J = \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{(pq)}{(p^2 - M_{\tilde{V}}^2)(p^2 - \mu^2)(q^2 - \mu^2)(q^2 - M_{\tilde{V}'}^2)(q^2 - m_i^2)(p^2 - m_j^2)} \frac{1}{[(p+q)^2 - M_{\Sigma}^2][(p+q)^2 - M_{\bar{\Sigma}}^2]} \quad (\text{C.3})$$

We can rewrite the fractions of (C.3) in a following way

$$\frac{1}{(p^2 - M_V^2)(p^2 - \mu^2)(p^2 - m_j^2)} = \sum_{n=1}^3 \frac{A_n}{p^2 - a_n^2}, \quad \frac{1}{(q^2 - M_{V'}^2)(q^2 - \mu^2)(q^2 - m_i^2)} = \sum_{m=1}^3 \frac{B_m}{q^2 - b_m^2},$$

$$\frac{1}{[(p+q)^2 - M_\Sigma^2][(p+q)^2 - M_\Sigma^2]} = \frac{1}{M_\Sigma^2 - M_\Sigma^2} \sum_{\alpha=1}^2 \frac{(-1)^{\alpha+1}}{(p+q)^2 - c_\alpha^2}$$

with $a_n^2 = (M_V^2, \mu^2, m_j^2)$, $b_m^2 = (M_{V'}^2, \mu^2, m_i^2)$, $c_\alpha^2 = (M_\Sigma^2, M_\Sigma^2)$ (C.4)

and

$$A_1 = \frac{1}{(M_V^2 - \mu^2)(M_V^2 - m_j^2)}, \quad A_2 = \frac{1}{(\mu^2 - M_V^2)(\mu^2 - m_j^2)}, \quad A_3 = \frac{1}{(m_j^2 - M_V^2)(m_j^2 - \mu^2)},$$

$$B_1 = \frac{1}{(M_{V'}^2 - \mu^2)(M_{V'}^2 - m_i^2)}, \quad B_2 = \frac{1}{(\mu^2 - M_{V'}^2)(\mu^2 - m_i^2)}, \quad B_3 = \frac{1}{(m_i^2 - M_{V'}^2)(m_i^2 - \mu^2)}. \quad (C.5)$$

Note the following properties of the coefficients:

$$\sum_{n=1}^3 A_n = \sum_{m=1}^3 B_m = 0, \quad \sum_{n=1}^3 A_n a_n^2 = \sum_{m=1}^3 B_m b_m^2 = 0. \quad (C.6)$$

Using (C.4), the integral in (C.3) can be written as a triple sum:

$$J = \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \sum_{\alpha,n,m} \frac{(-1)^{\alpha+1} A_n B_m}{M_\Sigma^2 - M_\Sigma^2} \frac{(pq)}{(p^2 - a_n^2)(q^2 - b_m^2)[(p+q)^2 - c_\alpha^2]}. \quad (C.7)$$

Moreover, using the identity

$$\frac{2(pq)}{(p^2 - a_n^2)(q^2 - b_m^2)[(p+q)^2 - c_\alpha^2]} = \frac{1}{(p^2 - a_n^2)(q^2 - b_m^2)} - \frac{1}{(q^2 - b_m^2)[(p+q)^2 - c_\alpha^2]} - \frac{1}{(p^2 - a_n^2)[(p+q)^2 - c_\alpha^2]} + \frac{c_\alpha^2 - a_n^2 - b_m^2}{(p^2 - a_n^2)(q^2 - b_m^2)[(p+q)^2 - c_\alpha^2]}, \quad (C.8)$$

the identities of Eq. (C.6) and $\sum_{\alpha=1}^2 (-1)^{\alpha+1} = 0$, from (C.7) we get:

$$J = \frac{1}{2(M_\Sigma^2 - M_\Sigma^2)} \sum_{\alpha,n,m} \gamma_{\alpha nm} \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{1}{(p^2 - a_n^2)(q^2 - b_m^2)[(p+q)^2 - c_\alpha^2]}$$

with $\gamma_{\alpha nm} \equiv (-1)^{\alpha+1}(c_\alpha^2 - a_n^2 - b_m^2)A_n B_m$. (C.9)

Using the Feynman parametrization

$$\frac{1}{d_1 d_2 d_3} = 2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{[x d_1 + y d_2 + (1-x-y) d_3]^3} \quad (C.10)$$

in the integral of Eq. (C.9), allows to perform integration with p and q . Due to the properties in (C.6) and $\sum_{\alpha=1}^2 (-1)^{\alpha+1} = 0$, the divergent parts disappear (as should be) and we remain with finite result given by:

$$J = -\frac{1}{2(16\pi^2)^2} \sum_{\alpha,n,m} \gamma_{\alpha nm} \int_0^1 dx \int_0^{1-x} dy C_{\alpha nm} \ln |C_{\alpha nm}|$$

with

$$C_{\alpha nm} = \frac{c_\alpha^2(x+y-1) - a_n^2 x - b_m^2 y}{(M_\Sigma^2 - M_\Sigma^2)(x^2 + y^2 + xy - x - y)^2} . \quad (\text{C.11})$$

Since the amplitude, defined in (C.3) (we have calculated), accounts for the operator $\frac{1}{4}(l^T \mathcal{M} l) h_u h_u$, we can identify the contribution to the neutrino mass matrix given in Eq. (38) [for definitions' references see also the comment after Eq. (38)].

References

- [1] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo and A. M. Rotunno, Phys. Rev. D **86** (2012) 013012; M. C. Gonzalez-Garcia, M. Maltoni and T. Schwetz, JHEP **1411** (2014) 052.
- [2] P. Minkowski, Phys. Lett. B **67** (1977) 421; M. Gell-Mann, P. Ramond and R. Slansky, Conf. Proc. C **790927** (1979) 315; T. Yanagida, Conf. Proc. C **7902131** (1979) 95; S. L. Glashow, NATO Sci. Ser. B **61** (1980) 687; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44** (1980) 912; J. Schechter and J. W. F. Valle, Phys. Rev. D **25** (1982) 774.
- [3] M. Magg and C. Wetterich, Phys. Lett. B **94** (1980) 61; J. Schechter and J. W. F. Valle, Phys. Rev. D **22** (1980) 2227; T. P. Cheng and L. F. Li, Phys. Rev. D **22** (1980) 2860; G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B **181** (1981) 287; R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23** (1981) 165.
- [4] R. Foot, H. Lew, X. G. He and G. C. Joshi, Z. Phys. C **44** (1989) 441; E. Ma, Phys. Rev. Lett. **81** (1998) 1171.
- [5] S. Weinberg, Phys. Rev. Lett. **43** (1979) 1566; Phys. Rev. D **22** (1980) 1694.
- [6] A. Zee, Phys. Lett. B **93** (1980) 389; Nucl. Phys. B **264** (1986) 99; K. S. Babu, Phys. Lett. B **203** (1988) 132.
- [7] A. Pilaftsis, Z. Phys. C **55** (1992) 275; K. S. Babu and J. Julio, Nucl. Phys. B **841** (2010) 130; Phys. Rev. D **85** (2012) 073005.
- [8] E. Ma, Phys. Rev. D **73** (2006) 077301.
- [9] P. S. B. Dev and A. Pilaftsis, Phys. Rev. D **86** (2012) 113001; S. Fraser, E. Ma and O. Popov, Phys. Lett. B **737** (2014) 280; S. Fraser, C. Kownacki, E. Ma and O. Popov, Phys. Rev. D **93** (2016) no.1, 013021; For a recent work and exhaustive list of references see: T. Nomura and H. Okada, arXiv:1609.01504 [hep-ph].

- [10] Y. Grossman and H. E. Haber, Phys. Rev. Lett. **78** (1997) 3438; N. Arkani-Hamed, L. J. Hall, H. Murayama, D. Tucker-Smith and N. Weiner, hep-ph/0007001; F. Borzumati and Y. Nomura, Phys. Rev. D **64** (2001) 053005; J. March-Russell and S. M. West, Phys. Lett. B **593** (2004) 181.
- [11] J. Wess and J. Bagger, “Supersymmetry and supergravity,” Princeton, USA: Univ. Pr. (1992) 259 pp.
- [12] G. F. Giudice and A. Masiero, Phys. Lett. B **206** (1988) 480.
- [13] J. Beringer *et al.* [Particle Data Group Collaboration], “Review of Particle Physics (RPP),” Phys. Rev. D **86** (2012) 010001.
- [14] B. Bajc, M. Nemevsek and G. Senjanovic, Phys. Rev. D **76** (2007) 055011; A. G. Akeroyd, M. Aoki and H. Sugiyama, Phys. Rev. D **77** (2008) 075010; P. Fileviez Perez, T. Han, G. y. Huang, T. Li and K. Wang, Phys. Rev. D **78** (2008) 015018. S. M. Boucenna, S. Morisi and J. W. F. Valle, Adv. High Energy Phys. **2014** (2014) 831598.
- [15] K. S. Babu, S. Nandi and Z. Tavartkiladze, Phys. Rev. D **80** (2009) 071702.
- [16] Z. Tavartkiladze, Phys. Lett. B **528** (2002) 97; Z. z. Xing and S. Zhou, Phys. Lett. B **679** (2009) 249; F. Bonnet, D. Hernandez, T. Ota and W. Winter, JHEP **0910** (2009) 076; S. Kanemura and T. Ota, Phys. Lett. B **694** (2011) 233; K. Kumericki, I. Picek and B. Radovic, Phys. Rev. D **86** (2012) 013006; P. W. Angel, N. L. Rodd and R. R. Volkas, Phys. Rev. D **87** (2013) no.7, 073007; and references therein.
- [17] S. Ferrara, M. Kaku, P.K. Townsend, P. van Nieuwenhuizen, Nucl. Phys. B **129** (1977) 125; M. Kaku, P. K. Townsend and P. van Nieuwenhuizen, Phys. Rev. D **17** (1978) 3179; E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, Nucl. Phys. B **147** (1979) 105.
- [18] T. Kugo and S. Uehara, Nucl. Phys. B **222** (1983) 125; V. Kaplunovsky and J. Louis, Nucl. Phys. B **422** (1994) 57.
- [19] J. Polonyi, Hungary Central Inst Res - KFKI-77-93 (77,REC.JUL 78) 5p.
- [20] A. E. Nelson and N. Seiberg, Nucl. Phys. B **416** (1994) 46.
- [21] D.Z. Freedman and A. Van Proeyen, “Supergravity,” Cambridge, Univ. Pr. (2012) 607 pp.
- [22] A. Brignole, Nucl. Phys. B **579** (2000) 101.